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Description of F_2 and the gluon at small x

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Abstract

We give a brief overview of the perturbative QCD description of the proton deep-inelastic structure function $F_2(x, Q^2)$ at small x. We discuss GLAP and BFKL approaches, and then we review progress towards a more unified treatment.

Résumé

Nous décrivons brièvement la fonction de structure $F_2(x, Q^2)$ du proton aux petites valeurs de x, dans l'approche perturbative de la chromodynamique quantique. Nous discutons les approches GLAP et BFKL, et passons en revue les progrès récents vers un traitement plus unifié.

1. Introduction

Fig. 1 is a sketch of the gluon content of the proton. GLAP or Altarelli-Parisi evolution to higher Q^2 increases the resolution $1/Q$ in the transverse plane, while BFKL evolution to small x builds up to gluon density until gluon recombination, or shadowing, can no longer be neglected. At small x the behaviour of $F_2(x, Q^2)$ follows that of the gluon on account of the $g \to q\bar{q}$ transition. First we briefly discuss the GLAP and BFKL descriptions of the measurements of $F_2(x, Q^2)$ at HERA, and then we review the progress made towards a unified treatment which incorporates both GLAP and BFKL dynamics.

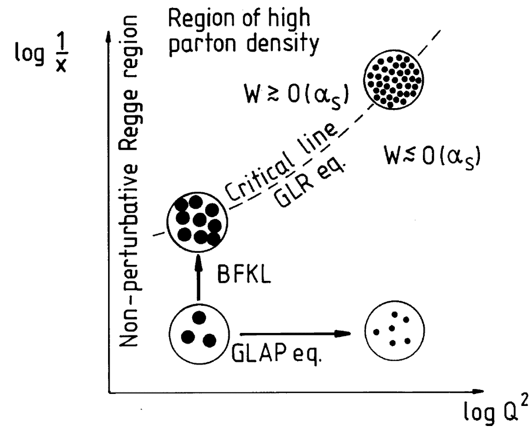

Figure 1. *The gluonic content of the proton as "seen" in different deep inelastic (x, Q^2) regions. The critical line, where gluon recombination becomes significant, occurs when $W \approx 0(\alpha_S)$. W is the ratio of the quadratic to the linear term on the right hand side of equation (9).*

2. GLAP description

There have been several successful attempts to describe the HERA data [1, 2] based on GLAP evolution [3, 4, 5, 6]. Fig. 2, which shows F_2 at $x = 4 \times 10^{-4}$, illustrates some of the main features. At $Q^2 = 4$ GeV^2, say, we have

$$F_2 \quad \sim \quad xS \sim A_S\, x^{-\lambda_S} \tag{1}$$

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$$\frac{\partial F_2}{\partial \log Q^2} \quad \sim \quad xg \sim A_g \, x^{-\lambda_g}. \tag{2}$$

That is, to a good approximation, F_2 determines the sea quark (S) distribution while the slope determines the gluon (g). The MRS(A$'$) and MRS(G) curves correspond to two global fits to deep-inelastic and related data. The former has $\lambda_g \equiv \lambda_S \simeq 0.2$, while in the latter both λ_g and λ_S are taken as free parameters with $\lambda_g \simeq 0.35$ and $\lambda_S \simeq 0.1$, that is a "steep" gluon and a "flat" sea distribution.

The GRV description [5] is obtained by evolving from valence-like parton distributions at a very low scale $Q_0^2 = 0.3$ GeV^2. By $Q^2 = 4$ GeV^2 the small x behaviour of xg and xS approximates the double leading logarithmic (DLL) form

$$\exp\left(2\left[\frac{36}{25}\log\left(\log\frac{Q^2}{\Lambda^2}/\log\frac{Q_0^2}{\Lambda^2}\right)\,\log\left(\frac{1}{x}\right)\right]^{\frac{1}{2}}\right). \tag{3}$$

Although not as "steep" as $x^{-\lambda}$, it can be approximated by this form over a limited interval of x. The effective value for GRV partons is $\lambda_g \simeq \lambda_S \simeq 0.28$, see Fig. 2. Ball and Forte [6] also find that the HERA F_2 data can be well described by DLL forms. In the DLL approach the effective slope λ can be decreased (increased) by simply increasing (decreasing) Q_0^2.

3. BFKL description

The BFKL description of F_2 at small x is based on the k_T-factorization formula [7]

$$F_2(x,Q^2) = \int_x^1 \frac{dx'}{x'} \int \frac{dk_T^2}{k_T^2}\, f(x',k_T^2)\, \hat{F}_2\left(\frac{x}{x'}, \frac{k_T^2}{Q^2}, \alpha_S\right) \tag{4}$$

where $f(x',k_T^2)$ is the gluon distribution *unintegrated* over k_T, while $\hat{F}_2$ is the structure function of a gluon of virtuality k_T^2 probed by a photon of virtuality Q^2, that is the contribution of the subprocess $\gamma g \to q\overline{q}$. f is calculated by integrating the BFKL equation

$$-\,x\,\partial f/\partial x = K \otimes f \tag{5}$$

down in x from a starting distribution at $x = x_0 = 0.01$, say. In this symbolic form of the equation $\otimes$ represents a convolution over k_T. The BFKL equation [8] effectively sums the leading $\alpha_S \log(1/x)$ contributions

$$f \sim \exp(\lambda \log(1/x)) \sim x^{-\lambda} \tag{6}$$

where λ represents the largest eigenvalue of the BFKL kernel K; $\lambda = 12\alpha_S \log 2/\pi$ for fixed α_S [8] and $\lambda \simeq 0.5$ for running α_S [9].

The solution of BFKL equation is sensitive to the treatment of the infrared (non-perturbative) region. For running α_S it is found that

$$f \sim C(k_T^2)x^{-\lambda} \tag{7}$$

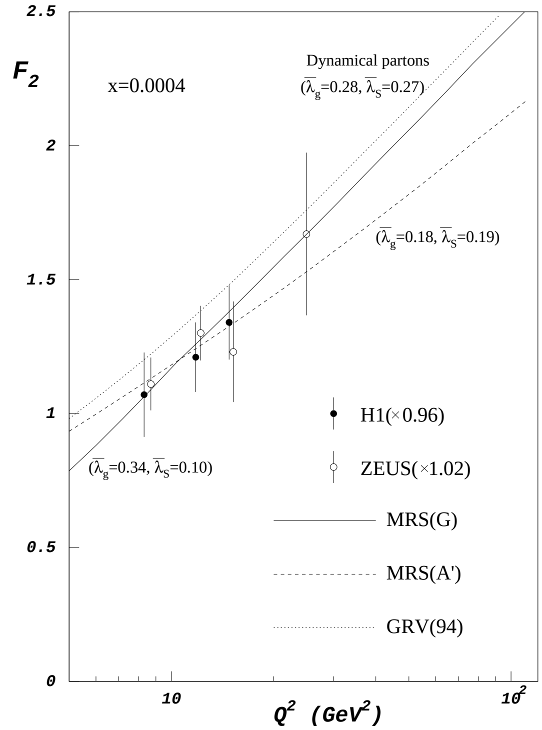

Figure 2. *HERA data [1, 2] for F_2^{ep} compared with MRS(A$'$,G) [3] and GRV [5] partons. The effective values of $\overline{\lambda}$, obtained from $xg \sim x^{-\overline{\lambda}_g}$ and $xS \sim x^{-\overline{\lambda}_S}$, are shown. The figure is taken from ref. [3].*

where $\lambda \approx 0.5$ has much less infrared sensitivity than the normalization C. The prediction for F_2 follows from the k_T-factorization formula (4)

$$\begin{aligned} F_2 \quad &= \quad f \otimes \hat{F}_2 + F_2^{bg} \\ &\simeq \quad C'(Q^2)x^{-\lambda} + F_2^{bg} \end{aligned} \tag{8}$$

where $\lambda \simeq 0.5$, and F_2^{bg} is determined from the large x behaviour of F_2. Once the overall normalization of the BFKL term is adjusted by a suitable choice of the infrared parameters a satisfactory description of the F_2 HERA data is obtained [9]. Indeed the BFKL-based treatment gives a similar description to GLAP. With GLAP, the observed steepness is either incorporated (as a factor $x^{-\lambda}$) in the starting distributions or generated by evolution from a low scale Q_0^2. The steepness can be adjusted to agree with the data by varying λ or Q_0^2. On the other hand the leading $\log(1/x)$ BFKL prediction for the shape $F_2 - F_2^{bg} \sim x^{-\lambda}$ with $\lambda \simeq 0.5$ is prescribed. It remains to see how well it survives a full treatment of sub-leading effects.

Shadowing is yet another possible ambiguity. The $x^{-\lambda}$ growth of the gluon cannot go on indefinitely with decreasing x. It would violate unitarity. The growth is

eventually suppressed by gluon recombination, which is represented by an additional quadratic term so that (5) has the form

$$- x\,\partial f/\partial x = K \otimes f - V \otimes f^2 \qquad (9)$$

where V contains a factor $\alpha_S^2/k_T^2 R^2$. The factor $1/R^2$ is expected; the smaller the transverse area (πR^2) in which the gluons are concentrated within the proton, the stronger the effect of recombination. If the gluons are spread uniformly throughout the proton ($R \sim 5\ \mathrm{GeV}^{-1}$) shadowing effects are predicted to be small in the HERA regime [9]. On the other hand if gluons are concentrated in "hot-spots" with, say, $R = 2\ \mathrm{GeV}^{-1}$ then shadowing gives an observable reduction in the prediction for F_2, particularly at low Q^2 [9]. In fact such a description is in line with the HERA data but, of course, other explanations are equally plausible.

In the remaining two sections we discuss the progress that is being made towards a unified description which incorporates both BFKL and GLAP evolution.

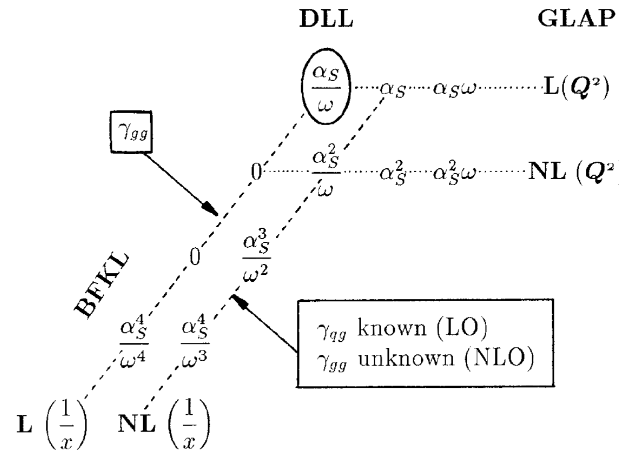

Figure 3. *Possible terms in the perturbative expansion of the anomalous dimensions (and associated splitting functions). Leading order GLAP and BFKL have only the DLL term in common.*

4. Perturbation series for $\gamma(\omega, \alpha_S)$

The possible onset of BFKL behaviour in the HERA small x domain has prompted several studies [10, 11, 12, 13] of the validity of GLAP evolution in this region. The situation is well summarised in Fig. 3 which shows the terms that occur in the expansion of the anomalous dimensions as power series in α_S and the moment index ω. Consider, for simplicity, GLAP evolution for the gluon alone

$$\frac{\partial g(x, Q^2)}{\partial \log Q^2} = \int_x^1 \frac{dy}{y}\, P_{gg}\left(\frac{x}{y}\right) g(y, Q^2). \qquad (10)$$

In moment space this takes the factorized form

$$\frac{\partial \overline{g}(\omega, Q^2)}{\partial \log Q^2} = \gamma_{gg}(\omega, \alpha_S)\, \overline{g}(\omega, Q^2)$$

where the anomalous dimension

$$\gamma_{gg} \equiv \int_0^1 dx\, x^\omega\, P_{gg}(x, \alpha_S) \approx \frac{\overline{\alpha}_S}{\omega}$$

if we use the small x approximation: $P_{gg} \approx \overline{\alpha}_S/x$ with $\overline{\alpha}_S \equiv 3\alpha_S/\pi$. This is the double leading logarithm (DLL) approximation, see Fig. 3. The terms that are included in full leading (and next-to-leading) order GLAP evolution are shown connected by horizontal dotted lines. On the other hand the BFKL equation resums a different subset of terms

$$\gamma_{gg} \simeq \sum_{n=1}^{\infty} A_n \left(\frac{\alpha_S}{\omega}\right)^n \to \sum_{n=1} A_n\, \alpha_S \frac{(\alpha_S \log 1/x)^{n-1}}{(n-1)!},$$

that is an ω^{-n} behaviour transforms into a $(\log 1/x)^{n-1}$ behaviour. Only these leading order $\log 1/x$ terms are known for γ_{gg}. Interestingly the coefficients $A_2 = A_3 = A_5 = 0$. Leading order for γ_{qg} corresponds to the sum of $\alpha_S(\alpha_S/\omega)^n$ terms, and here the coefficients are known. In fact the BFKL increase† of $F_2(\sim P_{qg} \otimes g)$ with decreasing x appears to be more due to the resummation in P_{qg} (and in the coefficient functions) than that for g.

Several numerical studies incorporating various $\alpha_S^n \omega^{-m}$ contributions have been undertaken [10, 11, 12, 13]. The perturbative QCD effects are unfortunately masked by the lack of knowledge of the non-perturbative input. In particular, the results are very dependent on the form of the 'starting' parton distributions. GLAP evolution from a singular $x^{-\lambda}$ input with $\lambda \gtrsim 0.3$ appears to be perturbatively stable. The situation for a 'flat' input is much more confused. Moreover care must be taken to avoid drawing definitive conclusions from a model dependent analysis in which the gluon and sea quark *input* behaviours are assumed to be strongly linked. The problem is that at small x we observe one structure function $F_2^{ep}(x, Q^2)$ and yet we need to freely parametrize both $g(x, Q_0^2)$ and $S(x, Q_0^2)$. A series of global analyses of deep inelastic data, systematically including more and more terms of Fig. 3, may be revealing.

† The reduction of the BFKL k_T-factorized form, (4), to the collinear form $F_2 \sim P_{qg} \otimes g$ is discussed in refs. [14, 15].

5. Unified CCFM equation

The CCFM equation [16] embodies both the BFKL equation at small x and GLAP evolution at large x. It is based on the coherent radiation of gluons, which leads to an angular ordering of gluon emissions. Outside the ordered region there is destructive interference between the emissions. For simplicity we concentrate on small x. Then the differential probability for emitting a gluon of momentum q is of the form

$$dP \sim \overline{\alpha}_S \Delta_R \frac{dz}{z} \frac{d^2 q_T}{\pi q_T^2}\, \Theta\left(\theta - \theta'\right) \qquad (11)$$

where successive gluon emissions occur at larger and larger angles. Δ_R represents the virtual corrections which screen the $1/z$ singularity. We can use (11) to obtain a recursion relation expressing the contribution of n gluon emission in terms of that of $n-1$. On summing, we find that the gluon distribution satisfies an equation

$$\begin{aligned} f(x, k_T^2, Q^2) \qquad &= f^0 + \overline{\alpha}_S \int_x^1 \frac{dz}{z} \int \frac{d^2 q}{\pi q^2} \Delta_R \; \times \\ & \Theta(Q - zq)\, f\left(\frac{x}{z}, |\boldsymbol{k}_T + \boldsymbol{q}|^2, q^2\right) \qquad (12) \end{aligned}$$

which we may call the CCFM equation [16]. The angular ordering introduces an additional scale (which turns out to be the hard scale Q of the probe), which is needed to specify the maximum angle of gluon emission.

When we unfold Δ_R, so that the real and virtual corrections appear on equal footing, and then take the leading $\log(1/x)$ approximation we find (12) reduces to the BFKL equation for a gluon distribution f which is independent of Q^2 (note that $\Theta(Q - zq) \to 1$). On the other hand in the large x region $\Delta_R \sim 1$ and $\Theta(Q - zq) \to \Theta(Q - q)$, which leads to GLAP transverse momentum ordering. If we replace $\overline{\alpha}_S/z$ by P_{gg} we see that (12) becomes the integral form of the GLAP equation.

Explicit solutions $f(x, k_T^2, Q^2)$ of the CCFM equation have recently been obtained in the small x region [17]. As anticipated, the CCFM equation generates a gluon with (i) a singular $x^{-\lambda}$ behaviour, with $\lambda \simeq 0.5$, (ii) a k_T distribution which broadens as x decreases and (iii) a suppression at low Q^2. Fig. 4 compares the effective λ of the integrated gluon, that is

$$xg(x, Q^2) \equiv \int^{Q^2} \frac{dk_T^2}{k_T^2}\, f(x, k_T^2, Q^2) \;\sim\; x^{-\lambda},$$

obtained from the CCFM solution, with that from the BFKL and DLL solutions. Both the CCFM and BFKL values converge to $\lambda \simeq 0.5$ at small x independent of Q^2, in contrast to DLL. In addition we see the angular ordering, embodied in the CCFM equation, leads to a suppression at low Q^2.

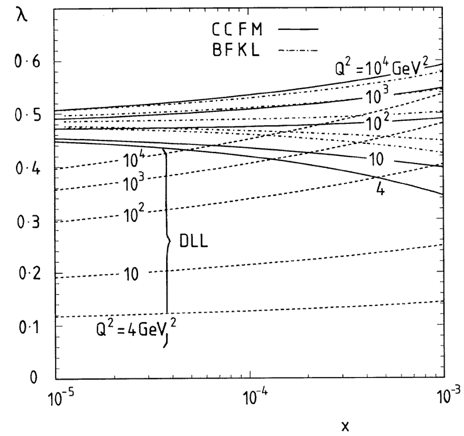

Figure 4. *The effective values of λ, defined by $xg = Ax^{-\lambda}$, obtained from the CCFM, BFKL equations and the conventional DLL approximation, for different values of Q^2. The figure is taken from ref. [17].*

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References

[1] H1 collab.: T. Ahmed et al., Nucl. Phys. **B439** (1995) 471.
[2] ZEUS collab.: M. Derrick et al., Z. Phys. **C65** (1995) 379.
[3] A. D. Martin, R. G. Roberts and W. J. Stirling, Durham preprint DTP/95/14 (Phys. Lett. in press)
W. J. Stirling, these proceedings.
[4] CTEQ collab.: H. L. Lai et al., Phys. Rev. **D51** (1995) 4763.
[5] M. Glück, E. Reya and A. Vogt, DESY 94-206 (Z. Phys. **C** in press).
[6] R. D. Ball and S. Forte, Phys. Lett. **B335** (1994) 77.
[7] S. Catani, M. Ciafaloni and F. Hautmann, Phys. Lett. **B242** (1990) 97; Nucl. Phys. **B366** (1991) 657.
[8] V. S. Fadin, E. A. Kuraev, L. N. Lipatov, Phys. Lett. **B60** (1975) 50; Sov. Phys. JETP **44** (1976) 433 and **45** (1977) 199; Ya. Ya. Balitskii and L. N. Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822.
[9] A. J. Askew, J. Kwiecinski, A. D. Martin and P. J. Sutton, Phys. Rev. **D47** (1993) 3775; **D49** (1994) 4402.
[10] R. K. Ellis, Z. Kunszt and E. M. Levin, Nucl. Phys. **B420** (1994) 517.
[11] R. K. Ellis, F. Hautmann and B. R. Webber, Phys. Lett. **B348** (1995) 582.
[12] R. D. Ball and S. Forte, Phys. Lett. **B351** (1995) 513.
[13] J. R. Forshaw, R. G. Roberts and R. S. Thorne, RAL report 95-035 (1995).
[14] S. Catani and F. Hautmann, Nucl. Phys. **B427** (1994) 475.
[15] J. Kwiecinski and A. D. Martin, Phys. Lett. **B353** (1995) 123.
[16] M. Ciafaloni, Nucl. Phys. **B296** (1988) 49;
S. Catani, F. Fiorani and G. Marchesini, Phys. Lett. **B234** (1990) 339; Nucl. Phys. **B336** (1990) 18;
G. Marchesini, these proceedings.
[17] J. Kwiecinski, A. D. Martin and P. J. Sutton, Phys. Rev. **D52** (1995) Aug. 1.